Lecture 9: The monetary theory of the exchange rate

Open Economy Macroeconomics, Fall 2006

Ida Wolden Bache

October 24, 2006

Macroeconomic models of exchange rate determination

- Useful reference: Chapter 4 of Sarno&Taylor (2002)

- Some influential classes of models
  - Portfolio balance models
  - The monetary approach
    - Flexible-price model
    - Sticky-price model
  - New Open Economy Macroeconomics

Portfolio balance models

- Useful reference: Branson&Henderson (1985) "*The specification and influence of asset markets*" in Handbook of International Economics, vol 2.

- Imperfect substitutability between different assets

- Wealth enters asset demand equations

The monetary approach

- Perfect substitutability between foreign and domestic assets (UIP)
- Exogenous money supply
- Flexible price models:
  - Prices respond instantly to shocks
  - Output is at its natural level at all times
  - Purchasing power parity (PPP) holds continuously ($P = SP^*$)

- Sticky price models (e.g., Dornbush (1976) overshooting model):
  - Prices respond sluggishly to shocks
  - Output deviates from natural level in the short run
  - Short-run deviations from PPP
  - Money neutral in the long run

New open economy macroeconomics (NOEM)

- Seminal contribution: Obstfeld&Rogoff (1995) "Exchange Rate Economics Redux", Journal of Political Economy vol 103

- Useful references: chapter 10 in Obstfeld&Rogoff (1996), chapter 5 in Sarno&Taylor (2002)

- Key features:
  - Dynamic general equilibrium model
  - Explicit microfoundations (intertemporally optimising agents)
  - Imperfect competition and nominal rigidities (sticky prices and/or sticky wages)
  - Changes in the money supply affect real variables in the short run

The monetary theory of the exchange rate

- Required readings: Chapter 4 in Rødseth (2000) and chapter 8.2.7 in Obstfeld&Rogoff (1996)

- Preliminaries on first-order linear differential equations (see e.g., ch 1 in Sydsæter et al. (2002) Matematisk Analyse Bind 2).

  - Define $\dot{x}(t) = dx/dt$

  - Consider the equation

  $$\dot{x}(t) + ax(t) = b(t)$$

  - For a given initial solution $x(t_0)$, the general solution is

  $$x(t) = x(t_0)\,\mathsf{exp}(-a(t-t_0)) + \int_{t_0}^{t} b(\tau)\,\mathsf{exp}(-a(t-\tau))d\tau$$

- Assumptions
  - International goods markets are perfectly integrated $\longrightarrow$ PPP holds continuously
  - Perfect capital mobility $\longrightarrow$ UIP
  - Rational expectations (perfect foresight)
  - Flexible prices and wages $\longrightarrow$ output is always at its natural level
  - Money supply is exogenous

- Focus on log-linearised model: lower case letters denote variables in natural logs ($x = \ln X$ and $\dot{x} = \dot{X}/X$)

- Model equations

$$
\begin{aligned}
p(t) &= s(t) + p^*(t) \\
\dot{s}(t) &= i(t) - i^*(t) \\
m(t) - p(t) &= -\eta i(t) + \kappa y(t),
\end{aligned}
$$

where p is the domestic price level, s is the nominal exchange rate, p^* is the foreign price level, m is the money supply, i is the nominal interest rate, i^* is the foreign nominal interest rate and y is the domestic output level

- Endogenous variables: s, p, i

- Exogenous variables: p^*, i^*, m, y

- Deriving the differential equation for the exchange rate

  1. Solve out for the domestic interest rate from the money market equilibrium condition

  $$i(t) = -\frac{1}{\eta}\left(m(t) - p(t) - \kappa y(t)\right)$$

  2. Substitute in for the domestic price level from the PPP condition

  $$i(t) = -\frac{1}{\eta}\left(m(t) - s(t) - p^*(t) - \kappa y(t)\right)$$

  3. Substitute into the UIP condition

  $$\begin{aligned}
\dot{s}(t) &= -\frac{1}{\eta}\left(m(t) - s(t) - p^*(t) - \kappa y(t)\right) - i^*(t) \\
\dot{s}(t) &= \frac{1}{\eta}s(t) - \underbrace{\frac{1}{\eta}\left(m(t) - p^*(t) - \kappa y(t)\right) - i^*(t)}_{z(t)} \\
\dot{s}(t) &= \frac{1}{\eta}s(t) - z(t)
\end{aligned}$$

- The differential equation for the exchange rate is fundamentally *unstable*: the higher is the level of the exchange rate, the higher is the rate of depreciation

- Intuition: $s \uparrow \longrightarrow p \uparrow$ (from PPP)$\longrightarrow m - p \downarrow \longrightarrow i \uparrow$ (from the money market equilibrium condition)$\longrightarrow \dot{s} \uparrow$ (from UIP)

- The exchange rate is free to jump to any level $\rightarrow$ without an initial condition there is an infinite number of solutions for the exchange rate

- The initial exchange rate is determined by the requirement that the exchange rate should tend to a strictly positive value when t goes to infinity

- Solution in the special case where the exogenous variables are expected to remain constant: exchange rate jumps immediately to the *stationary* value; i.e, the value corresponding to $\dot{s}(t) = 0$

$$\dot{s}(t) = 0 \Longrightarrow i = i^* \Longrightarrow s = m - p^* - \kappa y + \eta i^*$$

- General solution

$$
\begin{aligned}
s(t) &= s(t_0)\exp\left(\frac{1}{\eta}(t-t_0)\right) - \int_{t_0}^{t} z(\tau)\exp\left(\frac{1}{\eta}(t-\tau)\right)d\tau \\
&= s(t_0)\exp\left(\frac{1}{\eta}(t-t_0)\right) \\
&\quad - \int_{t_0}^{t} z(\tau)\exp\left(\frac{1}{\eta}(t-\tau) - \frac{1}{\eta}(t-t_0) + \frac{1}{\eta}(t-t_0)\right)d\tau \\
&= s(t_0)\exp\left(\frac{1}{\eta}(t-t_0)\right) \\
&\quad - \exp\left(\frac{1}{\eta}(t-t_0)\right)\int_{t_0}^{t} z(\tau)\exp\left(\frac{1}{\eta}(t-\tau) - \frac{1}{\eta}(t-t_0)\right)d\tau \\
&= \left[s(t_0) - \int_{t_0}^{t} z(\tau)\exp\left(-\frac{1}{\eta}\left(\tau-t_0\right)\right)d\tau\right]\exp\left(\frac{1}{\eta}\left(t-t_0\right)\right)
\end{aligned}
$$

- For the exchange rate not to explode the term in brackets must go to zero as t goes to infinity, i.e.,

$$s(t_0) = \int_{t_0}^{\infty} z(\tau) \exp\left(-\frac{1}{\eta}(\tau - t_0)\right) d\tau$$

- Recall that

$$\begin{aligned}
\int_a^b f(x)dx &= -\int_b^a f(x)dx \\
\int_a^b f(x)dx &= \int_a^c f(x)dx + \int_c^b f(x)dx
\end{aligned}$$

which implies

$$\int_a^b f(x)dx - \int_a^c f(x)dx = \int_c^a f(x)dx + \int_a^b f(x)dx = \int_c^b f(x)dx$$

- Insert into the general solution

$$
\begin{aligned}
s(t) &= \left[\begin{array}{c} \int_{t_0}^{\infty} z(\tau) \exp\left(-\frac{1}{\eta}(\tau - t_0)\right) d\tau \\ - \int_{t_0}^{t} z(\tau) \exp\left(-\frac{1}{\eta}(\tau - t_0)\right) d\tau \end{array} \right] \exp\left(\frac{1}{\eta}(t - t_0)\right) \\
&= \left[\int_{t}^{\infty} z(\tau) \exp\left(-\frac{1}{\eta}(\tau - t_0)\right) d\tau \right] \exp\left(\frac{1}{\eta}(t - t_0)\right) \\
&= \int_{t}^{\infty} z(\tau) \exp\left(-\frac{1}{\eta}(\tau - t_0) + \frac{1}{\eta}(t - t_0)\right) d\tau \\
&= \int_{t}^{\infty} z(\tau) \exp\left(-\frac{1}{\eta}(\tau - t)\right) d\tau \\
&= \int_{t}^{\infty} \left[\frac{1}{\eta}(m(\tau) - p^*(\tau) - \kappa y(\tau)) + i^*(\tau)\right] \exp\left(-\frac{1}{\eta}(\tau - t)\right) d\tau
\end{aligned}
$$

- The equilibrium exchange rate is the discounted sum of the whole (expected) future path of the exogenous variables where the discount rate is the inverse of the interest elasticity of money demand η.

- Qualifications
  - For the integral to converge z must grow at an absolute value smaller than $1/\eta$ as $t \to \infty$
  - The above solution is the *fundamental* solution, denoted $s_*(t)$. All solutions are related to $s_*(t)$ by

$$s(t) = s_*(t) + c \exp\left(\frac{1}{\eta} t\right),$$

    where c is a constant and there is one such solution for every real number c. The last term is a *rational bubble* and is unrelated to the underlying exogenous variables.

- When the exogenous variables are expected to remain constant the solution simplifies to

$$
\begin{aligned}
s(t) &= \left[\frac{1}{\eta}(m - p^* - \kappa y) + i^*\right] \int_t^\infty \exp\left(-\frac{1}{\eta}(\tau - t)\right) d\tau \\
&= \left[\frac{1}{\eta}(m - p^* - \kappa y) + i^*\right] \left[-\eta \exp\left(-\frac{1}{\eta}(\tau - t)\right)\right]_t^\infty \\
&= \left[\frac{1}{\eta}(m - p^* - \kappa y) + i^*\right] \left\{ \begin{array}{c} -\eta \underbrace{\exp\left(-\frac{1}{\eta}(\infty - t)\right)}_{0} \\ +\eta \underbrace{\exp\left(-\frac{1}{\eta}(t - t)\right)}_{1} \end{array} \right\} \\
&= m - p^* - \kappa y + \eta i^*
\end{aligned}
$$

- An unanticipated temporary increase in Δz
  - Assume that

$$z(t) = \begin{cases} z_0 + \Delta z & \text{if } t_0 < t < t_1 \\ z_0 & \text{if } t > t_1 \end{cases}$$

    where z_0 and Δz are positive constants

  - Then in period $t_0 < t < t_1$

$$\begin{aligned} s(t) &= \int_t^{\infty} z(\tau) \exp\left(-\frac{1}{\eta}(\tau - t)\right) d\tau \\ &= \int_t^{t_1} (z_0 + \Delta z) \exp\left(-\frac{1}{\eta}(\tau - t)\right) d\tau + \int_{t_1}^{\infty} z_0 \exp\left(-\frac{1}{\eta}(\tau - t)\right) d\tau \\ &= \underbrace{\int_t^{\infty} z_0 \exp\left(-\frac{1}{\eta}(\tau - t)\right) d\tau}_{s_0} + \int_t^{t_1} \Delta z \exp\left(-\frac{1}{\eta}(\tau - t)\right) d\tau \end{aligned}$$

– Recall that

$$\int \exp(ax)dx = \frac{1}{a}\exp(ax) + C$$

which implies

$$\begin{aligned} s(t) &= s_0 + \Delta z \left[-\eta \exp\left(-\frac{1}{\eta}(\tau - t) \right) \right]_t^{t_1} \\ &= s_0 + \Delta z \left(-\eta \exp\left(-\frac{1}{\eta}(t_1 - t) \right) + \eta \underbrace{\exp\left(-\frac{1}{\eta}(t - t) \right)}_{1} \right) \\ &= s_0 + \eta \left(1 - \exp\left(-\frac{1}{\eta}(t_1 - t) \right) \right) \Delta z \end{aligned}$$

– If $\Delta z = \frac{1}{\eta}\Delta m$

$$s(t) = s_0 + \left(1 - \exp\left(-\frac{1}{\eta}(t_1 - t) \right) \right) \Delta m$$

- Effect of a permanent change is found by letting $t_1 \to \infty$

$$s(t) = s_0 + \Delta m$$

Empirical evidence

- Weak empirical evidence for the monetary model as a model of short-run exchange rate movements

- Empirical tests tend to reject both
  - the joint hypothesis of UIP and rational expectations; and
  - the assumption of continuous PPP (see graph!)

- Some evidence supporting the model
  - in periods when inflation is high (e.g., Frenkel's (1976) study of the German hyperinflation in the 1920s)
  - as a model of 'long-run' exchange rate determination

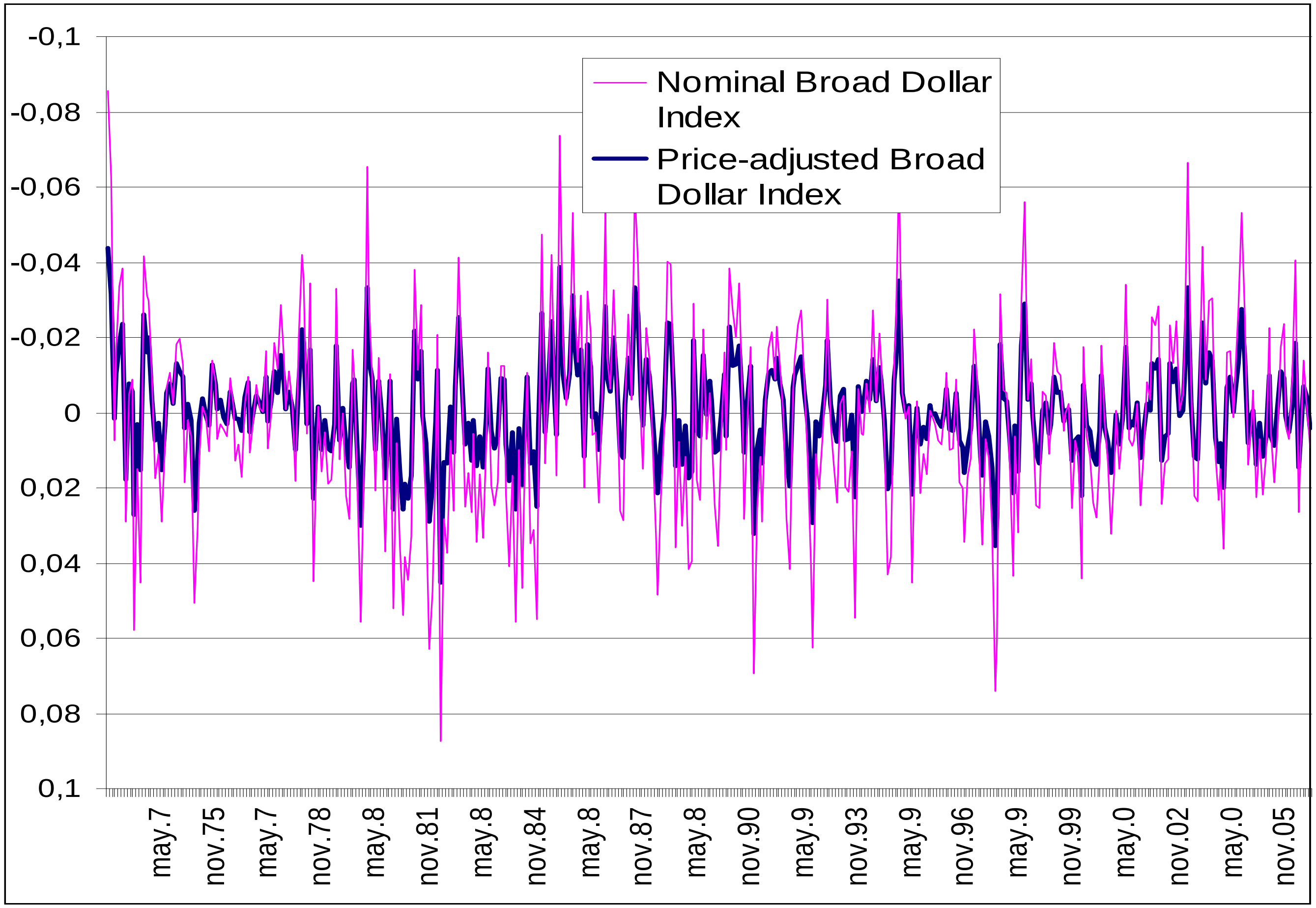

Figure 1: US nominal and real effective (trade-weighted) exchange rate (log differences)